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INTERFACIAL AREA AND NUCLEATION SITE DENSITY IN BOILING SYSTEMS

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Abstract—The number density, which is one of the most important parameters affecting the interfacial area concentration in the bubbly two-phase flow system, is formulated in terms of the differential transport equation. From this formulation it becomes apparent that the active nucleation site density on a heated channel surface is the key parameter in the prediction of the bubble number density. A constitutive relation for active nucleation site density is developed in pool boiling and extended to forced convective nucleate boiling. The active nucleation site densities predicted by this constitutive relation are in relatively good agreement with direct measurements available in the literature.

NOMENCLATURE

A_b	surface area of a typical bubble [m^2]
A_c	cross-sectional area [m^2]
a	thermal diffusivity [m^2 s^{-1}]
a_i	interfacial area concentration [m^{-1}]
C_p	specific heat at constant pressure [J kg^{-1} K^{-1}]
D	tube diameter [m]
D_d	bubble departure diameter [m]
D_{dF}	bubble departure diameter predicted by Fritz equation, $0.0208\theta(\sigma/g\Delta\rho)^{1/2}$ [m]
F	parameter defined by equation (36)
f	bubble generation frequency [s^{-1}]
G	mass velocity [kg m^{-2} s^{-1}]
h	heat transfer coefficient [W m^{-2} K^{-1}]
i	specific enthalpy [J kg^{-1}]
i_{fg}	heat of evaporation [J kg^{-1}]
k	thermal conductivity [W m^{-1} K^{-1}]
N_b	bubble number density [m^{-3}]
N_{cn}	active nucleation site density in forced convective nucleate boiling [m^{-2}]
N_{pn}	active nucleation site density in pool boiling [m^{-2}]
N^*	dimensionless active nucleation site density, ND_d^2 where $N = N_{cn}$ or N_{pn}
Nu	Nusselt number
P	pressure [N m^{-2}]
Pr	Prandtl number
$\dot{q}''$	heat flux [W m^{-2}]
$\dot{q}''_{boil}$	boiling component of surface heat flux [W m^{-2}]
$\dot{q}''_{cond}$	condensation heat flux at the outer edge of bubble boundary layer [W m^{-2}]
$\dot{q}''_{sp}$	single phase component of surface heat flux [W m^{-2}]
R_c	critical cavity size based on wall superheat [m]
R_{ce}	critical cavity size based on effective liquid superheat [m]
R_c^*	dimensionless cavity size, $R_c/(D_d/2)$
Re	Reynolds number
Re_{TP}	two-phase flow Reynolds number defined by equation (35)
r_s	surface equivalent radius [m]
r_{sm}	Souter mean radius [m]
r_v	volume equivalent radius [m]
S	suppression factor, defined by equation (34)
s	average distance between neighboring active nucleation sites [m]
T	temperature [K]
T_f	bulk fluid temperature [K]
T_0	bulk fluid temperature at the practical incipience boiling point [K]
T_{sat}	saturation temperature [K]
T_w	wall temperature [K]
t	time [s]
V_b	volume of a typical bubble [m^3]
v_{bz}	average bubble velocity [m s^{-1}]
$\bar{v}_f$	characteristic mean liquid velocity associated with a growing bubble [m s^{-1}]
X_{tt}	Martinelli parameter
x	quality
z	axial coordinate [m]

Greek symbols

α	void fraction
ΔT_{sat}	wall superheat, $T_w - T_{sat}$ [K]
η	fraction of number of bubbles that re-condense

θ	contact angle [deg.]
μ	dynamic viscosity [N s m^{-1}]
ξ_h	heated perimeter [m]
ρ_f	mass density of liquid phase [kg m^{-3}]
ρ_g	mass density of vapor phase [kg m^{-3}]
σ	surface tension [N m^{-1}]
ϕ	bubble generation rate, defined as the number of bubbles generated per unit volume per unit time [m^{-3} s^{-1}]
ϕ_{bn}	bubble nucleation rate in the bulk liquid [m^{-3} s^{-1}]
ϕ_{cond}	bubble sink rate [m^{-3} s^{-1}]
ϕ_{wn}	bubble nucleation rate from active cavities [m^{-3} s^{-1}]
ϕ_{dis}	bubble generation rate due to disintegration [m^{-3} s^{-1}]

Subscripts

f	liquid phase
g	vapor phase
w	wall
cn	forced convection nucleation
pn	pool boiling nucleation

Superscripts

*	dimensionless quantities

1. INTRODUCTION

IN PREDICTING two-phase flow transients, the interfacial transfer terms are among the most essential factors in the modeling. These interfacial transfer terms in a two-fluid model specify the rate of phase change, momentum exchange and heat transfer at the interface between phases.

A two-fluid model [1–4] is formulated in terms of two sets of conservation equations governing the balance of mass, momentum and energy of each phase. Since the macroscopic fields of one phase are not independent of those of the other phase, the interaction terms which couple the transport of mass, momentum and energy of each phase across the interfaces appear in the field equation [1]. In the two-fluid model formulation, the transport processes of each phase are expressed by their own balance equations. Therefore, it is expected that the model can predict more detailed changes and phase interactions than a mixture model such as the drift flux model [5, 6]. In particular, for two-phase flow problems involving a sudden acceleration of one phase, inertia terms of each phase should be considered separately by use of a two-fluid model.

The weakest link in the two-fluid model formulation is the constitutive equations for the interfacial interaction terms. The difficulties arise due to the complicated motion and geometry of the interfaces in a general two-phase flow. Furthermore, these constitutive equations should be expressed by the macroscopic variables based on proper averaging. As has been shown in detail [1, 7], the interfacial transfer terms in a two-fluid model appear as averaging of local instant transfers of mass, momentum and energy. Because these terms appear as source terms in the field equations, proper averaging alone is not sufficient to develop these constitutive equations. It is essential, therefore, to clarify the different physical mechanisms controlling these interfacial transfers as well as to identify the important parameters which govern them.

The interfacial transfer terms are strongly related to the interfacial area and to the local mechanisms, such as the degree of turbulence near the interfaces [1]. Basically, the interfacial transport of mass, momentum and energy is proportional to the interfacial area concentration and to a driving force. This area concentration, defined as the interfacial area per unit volume of the mixture, characterizes the kinematic effects; therefore, it must be related to the structure of the two-phase flow field. On the other hand, the driving forces for the interphase transport characterize the local transport mechanism, and they must be modeled separately.

Basic macroscopic parameters related to the structure of two-phase flows, particularly of a dispersed (bubbly or droplet) flow, are the void fraction, particle number density, interfacial area concentration and the particle shape factor. From geometric considerations, it is demonstrated that the particle number density is a key parameter in determining the interfacial area concentration but it has not been sufficiently investigated in the literature.

Realizing the significance of the bubble number density as an important parameter for predicting the interfacial area in a forced convective nucleate boiling channel, the following are the objectives of this paper: (i) to formulate the bubble number density in terms of the differential balance equation which takes into account various parameters such as the bulk liquid nucleation, the wall cavity nucleation and the bubble collapse rates through the source and sink terms, (ii) to discuss the numerical importance of these parameters in the subcooled, as well as the saturated nucleate boiling regions, (iii) to develop an empirical correlation for the heated surface cavity nucleation rate which can be used as a constitutive equation in the bubble number density transport equation.

2. INTERFACIAL AREA FORMULATION

The interfacial area concentration in a heated boiling channel can be formulated if the complete size spectrum of bubbles is available. However, the basic experimental data needed to go in this direction are grossly inadequate at the present time. Consequently, it does not seem appropriate to start from a given size distribution of bubbles, but formulate instead the interfacial area concentration by means of the basic macroscopic parameters related to the structure of the bubbly two-phase flow [8]. These are the Souter mean radius, $r_{sm} \equiv 3V_b/A_b$, the volume equivalent radius, $r_v \equiv (3\,V_b/4\pi)^{1/3}$, and the surface equivalent radius, $r_s = (A_b/4\pi)^{1/2}$. These characteristic radii represent various length scales for a bubbly two-phase flow. Other parameters are the void fraction, α, and the bubble number density, N_b, defined as the number of bubbles per unit volume of the mixture. These two parameters are related to each other by $\alpha = N_b V_b$ and to the interfacial area concentration, a_i, by

$$a_i = N_b A_b = \alpha(A_b/V_b) \quad (1)$$

where A_b and V_b, respectively, are the surface area and the volume of a typical bubble.

Using the above definitions the area concentration can be expressed in a number of forms. For example,

$$a_i = 3\,\alpha/r_{sm} = 3(\alpha/r_v)(r_s/r_v)^2 = 3(\alpha/r_v)(r_v/r_{sm}) \quad (2)$$

or

$$a_i = 4.84(r_s/r_v)^2 N_b^{1/3}\alpha^{2/3} \quad (3)$$

where (r_s/r_v) and (r_v/r_{sm}) are important shape factors which relate various length scales at interfaces. It is evident from the definitions that for spherical bubbles, $r_s/r_v = r_v/r_{sm} = 1$. However, the deviations of these shape factors from unity become significant as deformations of bubbles increase.

For an adiabatic flow, equation (2) is useful because the size of the particles may be determined from initial and boundary conditions. For a two-phase flow with phase changes, equation (3) is more convenient because the size of bubbles change due to phase changes. Therefore, concentrating on equation (3) it can be observed that in order to predict the interfacial area concentration in bubbly two-phase flow in a heated channel, it is necessary to have information about α, (r_s/r_v) and N_b. Among these parameters, the void fraction is one of the dependent variables in a two-fluid model formulation and must be determined from the field equations, and the shape factor has been studied and correlated by the Eotvos number [8]. However, the third parameter, N_b, has not been sufficiently investigated in the literature.

3. BUBBLE NUMBER DENSITY FORMULATION

Considering a boiling channel with a constant cross-sectional area, a 1-dim. transport equation for predicting the average bubble number density (averaged over a cross-sectional area of the channel) can be expressed as

$$\frac{\partial N_b}{\partial t} + \frac{\partial}{\partial z}(N_b v_{bz}) = \phi_{bn} + \phi_{wn} + \phi_{dis} - \phi_{cond} \quad (4)$$

where the symbols are defined in the nomenclature.

It is evident from equation (4) that three effects must be taken into account in an analysis concerned with predicting the bubble number density. These are: (1) the bulk liquid bubble nucleation rate, ϕ_{bn}; (2) the heated wall cavity nucleation rate, ϕ_{wn}; (3) the bubble number density sink rate due to bubble coalescence and collapse, ϕ_{cond}; and (4) the bubble number density generation rate due to bubble disintegration, ϕ_{dis}. It is to be noted that the disintegration of bubbles in a nucleate boiling channel is not as important as the other terms. However, it may be significant for large bubbles in a highly turbulent flow. We shall discuss the significance of the first three variables and the way they can be accounted for in the analysis.

3.1. *Bulk liquid nucleation*

In general, bubble nucleation in the bulk liquid may be either of the homogeneous or heterogeneous types. Designating the homogeneous and heterogeneous nucleation rates by ϕ_{ho} and ϕ_{he}, respectively, one can write

$$\phi_{bn} = \phi_{ho} + \phi_{he}. \quad (5)$$

Several theories based on statistical mechanics have been proposed to account for homogeneous nucleation in the pure liquid. One approach using classical rate theory [9] presumes that numerous molecules have the activation energy required for existence in the vapor phase. These energetic molecules combine through collisions to form a cluster, which is then a vapor bubble. Theories of this type yield extremely high superheats for nucleation in a pure liquid. Such high superheats are contrary to experimental observations with real systems. Therefore, homogeneous nucleation can be discounted as a main mechanism for bubble formation in a heated channel.

In a real system, the liquid contains foreign particles and dissolved gas, which could act as nuclei. The predicted nucleation superheats would be considerably less in the presence of a pre-existing dissolved gas. This form of heterogeneous nucleation, ϕ_{he}, implies that vapor formation would be noted at random points where the nucleii happen to be located. This type of bubble generation may become important when significant depressurization occurs within a small fluid residence time. For example, it could be a significant process during the discharge of flashing steam–water mixtures through short nozzles or orifices [10].

3.2. *Heated surface nucleation*

In a system with a heat addition, bubbles form at cavities on the heated surface known as nucleation sites. Therefore, the surface nucleation rate, ϕ_{wn}, must be related to the number of nucleation sites that are activated at a given liquid superheat. Introducing N_{cn}, the active nucleation site density in convective nucleate boiling, and f, the bubble generation frequency from active sites, one can express ϕ_{wn} as follows:

$$\phi_{wn} = (N_{cn} f \xi_h)/A_c \quad (6)$$

where ξ_h and A_c are the heated perimeter and the cross-sectional area of the boiling channel, respectively. Equation (6) indicates, therefore, that the active nucleation site density is an important parameter in determining the surface nucleation rate in a boiling channel. Therefore, a detailed modeling of the active nucleation site density will be carried out in the following sections.

3.3. *Bubble number sink rate*

In general, reduction in bubble number density, which was taken care of by the sink term, ϕ_{cond}, may be due to either coalescence of bubbles into a larger bubble or re-condensation of bubbles in the subcooled bulk fluid. Here the effort has been concentrated on determining the re-condensation rate of generated bubbles. The coalescence is assumed to be insignificant up to the void fraction of 0.3 beyond which the flow regime transition to the slug or churn-turbulent flow occurs.

The highly subcooled boiling region of a flow channel is of little significance as far as the net bubble generation is concerned, and in this region $\phi_{\text{wn}} \simeq \phi_{\text{cond}}$. For all purposes, therefore, the transition point which has been studied in the literature [11–14] can be regarded as the point of net bubble generation. The fraction or re-condensation in the region downstream of the transition point can be obtained by comparing the rates of net vapor generation and evaporation at the surface. In order to make an estimate of these two rates, we can assume that the rate of evaporation at the surface will be proportional to the total heat flux minus the single phase convective heat flux, whereas the rate of net vapor formation will be proportional to the amount of energy that is used to increase vapor flow. Consequently the fraction of bubbles that re-condense, η, can be estimated as follows:

$$\eta = 1 - \frac{\mathrm{d}}{\mathrm{d}z}\{G_g[i_{fg} + C_{pf}(T_{\text{sat}} - T_f)]\}/(\dot{q}'' - \dot{q}''_{\text{sp}})(\xi_h/A_c) \tag{7}$$

where G_g is the vapor mass velocity, T_f is the liquid bulk temperature and $\dot{q}''_{\text{sp}}$ is the single phase convective heat flux. In terms of η, ϕ_{cond} can be expressed by

$$\phi_{\text{cond}} = \eta\phi_{\text{wn}}. \tag{8}$$

With constant properties, using the mixture energy and mass balance equations on the RHS of equation (7), it can be shown that

$$\eta = \frac{GC_{pf}(\mathrm{d}T_f/\mathrm{d}z) - \dot{q}''_{\text{sp}}(\xi_h/A_c)}{(\dot{q}'' - \dot{q}''_{\text{sp}})(\xi_h/A_c)}. \tag{9}$$

It is important to note that the expression derived for the prediction of η satisfies two limiting cases. Until the transition point is reached, the total energy supplied to the subcooled liquid is used for increasing the temperature of the bulk liquid without net vapor generation. Therefore, in this region η approaches unity in equation (9). On the other hand, in the saturated nucleate boiling region $\mathrm{d}T_f/\mathrm{d}z = 0$, and $\dot{q}''_{\text{sp}}$ is insignificant when compared with $\dot{q}''$. Therefore, η becomes zero in equation (27), indicating that the rate of re-condensation is zero.

It is evident from equation (9) that in order to evaluate η, it is necessary to determine first the axial liquid bulk temperature distribution, $T_f(z)$. In refs. [15–17], the axial temperature distribution for subcooled boiling has been approximated by functions that satisfy the physical boundary conditions. It was shown in these references that the exponential or the hyperbolic tangent approximations for the temperature profile gave satisfactory results when compared with experimental data. When the exponential temperature profile is used in equation (9), η can be expressed as follows:

$$\eta = \frac{[(T_{\text{sat}} - T_f)/(T_{\text{sat}} - T_0)] - (\dot{q}''_{\text{sp}}/\dot{q}'')}{1 - (\dot{q}''_{\text{sp}}/\dot{q}'')} \tag{10}$$

where T_0 is the bulk liquid temperature at the transition point.

This expression is general if we use the standard single phase heat transfer correlation for $\dot{q}''_{\text{sp}}$ and two-phase flow heat transfer correlation. However, this will lead to a highly complicated correlation for ϕ_{cond}. Therefore, it is desirable to obtain a simple expression which satisfies the overall physical phenomenon.

First, consider the fully developed subcooled boiling region where $\dot{q}''_{\text{sp}}$ is insignificant in comparison with nucleate boiling heat flux. Under this approximation, equation (10) reduces to

$$\eta = (T_{\text{sat}} - T_f)/(T_{\text{sat}} - T_0). \tag{11}$$

This relation satisfies the limiting condition of saturated boiling. Since η approaches to zero when T_f approaches to T_{sat}, this indicates that the amount of re-condensation becomes zero.

Second, consider the neighborhood of the transition point in the relatively highly subcooled boiling region. In this region, the single phase convection heat transfer is significant relative to the nucleate boiling heat transfer. Then in equation (10) $\dot{q}''_{\text{sp}}$ should play a major role in determining η or the re-condensation rate. However, as T_f approaches to T_0, $\phi_{\text{cond}} \to \phi_{\text{wn}}$ because all bubbles nucleated should immediately re-condense. This implies that $\eta \to 1$ as $T_f \to T_0$.

Now in terms of the local heat flux η can be also expressed as

$$\eta = \dot{q}''_{\text{cond}}/\dot{q}''_{\text{boil}} \tag{12}$$

where $\dot{q}''_{\text{boil}}$ is the nucleate boiling component of the wall heat flux and $\dot{q}''_{\text{cond}}$ is the condensation heat flux at the outer edge of the bubble boundary layer. Since $\dot{q}''_{\text{cond}}$ is mainly governed by the single phase heat transfer in the highly subcooled liquid core where a steep liquid temperature gradient exists outside the bubble boundary layer it is expected that $\dot{q}''_{\text{cond}} \sim (T_{\text{sat}} - T_f)$.

The limiting condition of η at $T_f = T_0$ and equation (12) imply that the approximate expression for η in this region should be as follows:

$$\eta \simeq (T_{\text{sat}} - T_f)/(T_{\text{sat}} - T_0). \tag{13}$$

It is interesting to note that this equation has the same form as equation (11). Therefore, from these two limiting conditions it may be concluded that the general expression for η can be given approximately by equation (11).

In view of equations (8) and (11), the sink term can be expressed as

$$\phi_{\mathrm{cond}} = [(T_{\mathrm{sat}} - T_{\mathrm{f}})/(T_{\mathrm{sat}} - T_0)]\phi_{\mathrm{wn}} \quad \text{for} \quad T_0 \leqslant T_{\mathrm{f}} \leqslant T_{\mathrm{sat}}. \tag{14}$$

Combining the bubble number density sink term, equation (14), and the surface nucleation rate term expressed by equation (6) and substituting the resulting equation in equation (4), the bubble number density transport equation can be expressed as

$$\frac{\partial N_{\mathrm{b}}}{\partial t} + \frac{\partial}{\partial z}(N_{\mathrm{b}} v_{\mathrm{b}z}) = \left(\frac{T_{\mathrm{f}} - T_0}{T_{\mathrm{sat}} - T_0}\right)\left(\frac{N_{\mathrm{cn}} f \xi_{\mathrm{h}}}{A_{\mathrm{c}}}\right) + \phi_{\mathrm{he}} + \phi_{\mathrm{dis}}. \tag{15}$$

In view of the foregoing discussion, the major contribution to the bubble source is due to the wall cavity nucleation in a heated channel. It appears as the first term on the RHS of equation (15). Therefore, it is desirable to develop a constitutive relation for the active nucleation site density which can be used in the bubble number density transport equation. This will be done in the sections that follow.

4. ACTIVE NUCLEATION SITE DENSITY IN POOL BOILING

4.1. *Pool boiling data*

Starting from the fact that there exists a mechanistic similarity in cavity nucleation between pool boiling and convective nucleate boiling, active nucleation site density studies are based on the pool boiling experimental results. It has been generally agreed [18–21] that the pool boiling active nucleation site density, N_{pn}, can be determined as a function of the cavity radii in the form

$$N_{\mathrm{pn}} = C R_{\mathrm{c}}^{m} \tag{16}$$

where R_{c} is the minimum cavity radius at a specified condition which is defined by

$$R_{\mathrm{c}} = 2\sigma T_{\mathrm{sat}}/\rho_{\mathrm{g}} i_{\mathrm{fg}} \Delta T_{\mathrm{sat}} \tag{17}$$

where C and m are constants characterizing the boiling surface.

For practical purposes, there are difficulties in using equation (16) at this time because of the following reasons: (1) Since the essential elements of a surface can not be measured and related mathematically to boiling performance, with a new surface a new set of experiments has to be run to determine the surface parameters C and m. (2) Derivation of equation (17) involves certain assumptions concerning the fluid properties, and it was demonstrated [22] that it may lead to significant errors when it is used for a wide range of pressures. (3) Although a number of pool boiling data providing quantitative information on the active nucleation site density have been accumulated over the years, none of the experimenters when investigating the effect of pressure on boiling performance simultaneously counted active nucleation sites; therefore, the pressure ranges covered by these data are far short of being sufficient to warrant the use of equation (16).

Starting from these considerations, in the present investigation a global approach was followed to correlate the active nucleation site density to measurable quantities. Collecting a large number of data from different sources the surface effects are smoothed out. This overall approach certainly eliminates the detailed effects of the surface characteristics on the nucleation process, but it will take into account these effects in an averaged sense. Due to the lack of quantitative data on the number N_{pn} over a wide range of pressure, first, the heat transfer coefficient is correlated to the liquid superheat and the active nucleation site density. Then this correlation is used to obtain quantitative N_{pn} data from experimentally measured values of liquid superheats and heat transfer coefficients.

4.2. *Heat transfer correlation*

In order to model the heat transfer in nucleate boiling, it is important to describe the hydrodynamic field of the fluid adjacent to the heated surface. For boiling on a horizontal surface, for example, the liquid flow pattern in the vicinity of a nucleation center continuously oscillates between source flow and wake flow [23]. The source flow is associated with the bubble growth in a superheated liquid during the growth period, whereas the wake flow is associated with the departure of bubbles and their rise through the superheated liquid during the waiting period. Therefore, these two flow patterns which follow each other in a cycle must be taken into account in a formal analysis concerned with predicting the heat transfer in nucleate boiling.

There are difficulties in following this formal approach at this time because of the uncertainties related to: (1) the growth period which depends on the local superheat and the local hydrodynamic conditions, and (2) the waiting period which depends upon the local heat flux, local thermal fluctuations in the liquid and the nucleation center size. At this point none of these parameters is known with sufficient accuracy to warrant the use of the formal approach to obtain a heat transfer correlation. Instead of making several assumptions pertaining to these parameters, only one assumption is introduced here by adopting the source flow model. In addition to its simplicity, this model has two advantages at present. First, a unique expression can be used to describe both subcooled and saturated nucleate boiling because the degree of agitation of the source flow in subcooled boiling is independent of the degree of subcooling itself. Second, a correlation based on the source flow model can be applied to boiling on both horizontal and vertical surface because the local agitating effect of the source flow is similar in both cases.

In view of the proposed source flow model, the liquid motion in the vicinity of a nucleation center is approximated by a radial motion within a characteristic dimension of $s/2$, where s is the average distance between two neighboring active nucleation centers. While it is known that the active nucleation sites are distributed rather randomly over a heated surface, s is directly related to active nucleation site density [23], and is given by

$$s = N_{pn}^{-1/2}. \tag{18}$$

The heat transfer data gathered from a wide variety of sources were fitted with the equation

$$Nu = \text{const.}\ Re^{a} Pr^{b} N_{pn}^{*c}. \tag{19}$$

The Nusselt number is defined by

$$Nu = h/k_f N_{pn}^{1/2} \tag{20}$$

whereas the Reynolds number is defined as

$$Re = \rho_f \bar{v}_f/\mu_f N_{pn}^{1/2} \tag{21}$$

where $\bar{v}_f$ is the characteristic mean liquid velocity associated with a growing bubble, given by ref. [23] as

$$\bar{v}_f = 1.27(\rho_f C_{pf} a_f^{1/2} \Delta T_{sat}/\rho_g i_{fg})^2 N_{pn}^{1/2}. \tag{22}$$

The dimensionless quantity N_{pn}^*, which scales the bubble departure diameter with the growing bubble influence domain, is defined as

$$N_{pn}^* = N_{pn} D_d^2 \tag{23}$$

where D_d is the bubble departure diameter. It should be noted that this dimensionless group is limited by unity because in the maximum packing condition $N_{pn}D_d^2 \leqslant 1.0$. In almost all of the boiling heat transfer studies, the Fritz equation has been used for D_d. However, a comparison of the Fritz equation with the available experimental water data has conclusively shown that the Fritz equation yields a good agreement only around atmospheric pressure [22]. For higher pressures, the Fritz equation is modified on experimental water data and the following expression is used:

$$D_d = 0.0012(\Delta\rho/\rho_g)^{0.9} D_{dF} \tag{24}$$

where D_{dF} is the bubble departure diameter calculated by the Fritz equation.

Fitting the experimental data gathered from a wide variety of sources [25–27] with equation (19) resulted in the following correlation:

$$h/(k_f N_{pn}^{0.5}) = 14.0[(\rho_f C_{pf} \Delta T_{sat})/(\rho_g i_{fg})]^{0.5} Pr^{-0.39} N_{pn}^{*\,-0.125}. \tag{25}$$

Comparisons of predicted results with the experimental values are presented in Figs. 1 and 2 in dimensional form. Considering the variety of surfaces and fluids used in experiments, equation (25) correlates the data well over a wide range of variables.

It is interesting to note that the functional relationship between the heat transfer coefficient and the basic parameters is of the form

$$h = \text{const.}\ (\Delta T_{sat})^{0.5} N_{pn}^{0.375}. \tag{26}$$

The form of equation (26) is confirmed by the experimental observations where it has been shown that the heat transfer coefficient is not a single-valued function of the superheat, but depends upon both the superheat and the active nucleation site density. Equation (25) will be used in the development of the active nucleation site density correlation for water to obtain quantitative data from experimental values of h and ΔT_{sat}.

4.3. *Active nucleation site density correlation*

When attempting a general correlation of many experimental data, one can build up a mechanistic model, derive the general form of an equation from it, and adopt the constants and exponents in this equation to the experimental data. However, the present knowledge on boiling and, in particular, on surface nucleation characteristics is not sufficient for building up a valid general model to correlate the active nucleation site density, N_{pn}. Until the time when the essential elements of a surface can be measured and are related mathematically to boiling performance, it is unlikely that any correlation based on mechanistic modeling can be developed for determining N_{pn}. Consequently, it is decided to correlate the existing experimental data by means of parametric study.

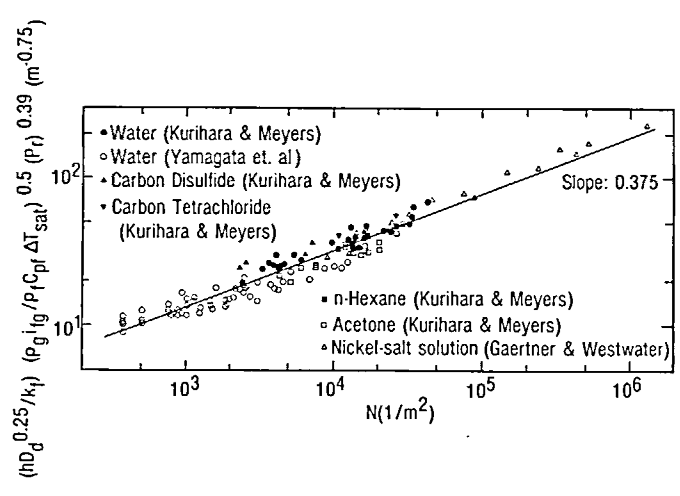

FIG. 1. Correlation of heat transfer coefficient with active nucleation site density.

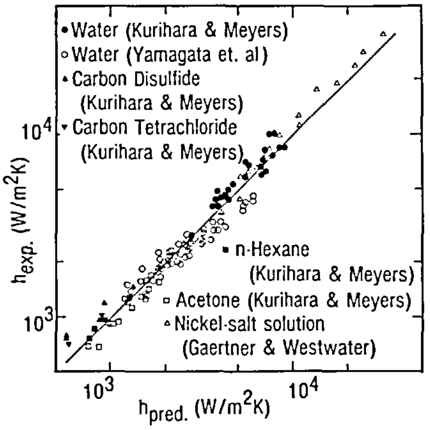

FIG. 2. Comparison of predicted heat transfer coefficient with experimental values.

It is assumed that N_{pn} is influenced by both the surface conditions and the thermophysical properties of the fluid. For a given fluid, properties may be described as a function of pressure only, and realizing the significance of the minimum cavity size as an important surface parameter, it is proposed that N_{pn} can be expressed as

$$N_{pn} = N_{pn}(R_c, P). \tag{27}$$

Using the bubble departure diameter as scaling parameter for N_{pn} and R_c, in dimensionless form equation (27) becomes

$$N^*_{pn} = N^*_{pn}(R^*_c, \rho^*) \tag{28}$$

where the dimensionless group $\rho^* = (\rho_f - \rho_g)/\rho_g$ represents the pressure dependency and the dimensionless group R_c is defined as $R^*_c = R_c/(D_d/2)$.

Combining the Thomson equation for the excess pressure with the Clausius–Clapeyron relation, it can be shown that the minimum cavity size which can be activated at a given superheat is given by

$$R_c = \{2\sigma[1+(\rho_g/\rho_f)]/P_f\} \times \{\exp[i_{fg}(T_g - T_{sat})/(RT_gT_{sat})] - 1\} \tag{29}$$

where R is the ideal gas constant. It should be noted that if $\rho_g \ll \rho_f$ and $[i_{fg}(T_g - T_{sat})/(RT_gT_{sat})] \ll 1$ only then equation (29) simplifies to equation (17). These two approximations may be satisfied simultaneously only in a moderate pressure range. As it was quantitatively demonstrated [22], the values of R_c predicted by equation (17) considerably differ from those predicted by equation (29) at relatively low and high pressures. Since it is the purpose of this work to correlate the experimental data for a wide range of pressures, equation (29) has been used throughout.

Before attempting a general correlation of many water data from different sources, a parametric study was conducted first. Equations (25) and (29) are used to calculate N_{pn} and R_c, respectively, from experimentally measured values of ΔT_{sat}, h and from thermophysical properties of the water. Note that for the pool boiling $T_g - T_{sat} \simeq \Delta T_{sat}$ is used in equation (29). As illustrated in Fig. 3, this study indicates that the arbitrary function proposed by equation (28) may be replaced for water by

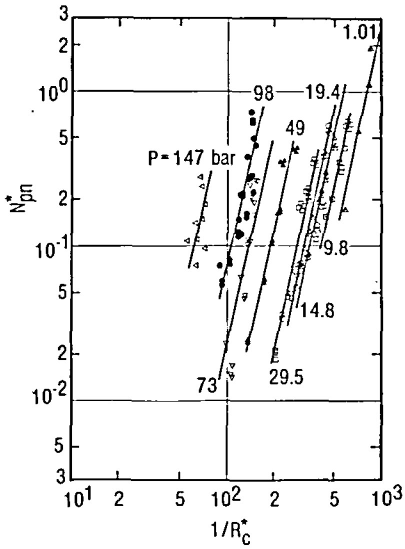

FIG. 3. Dimensionless active nucleation site density vs dimensionless critical cavity size at different pressures. (Calculated from experimental data of ref. [28].)

$$N^*_{pn} = R^{*m}_c f(\rho^*). \tag{30}$$

Finally, fitting the water data [25, 26, 28–31] with equation (30), the exponent m, and the function $f(\rho^*)$, were determined. The final result can be represented by

$$N^*_{pn} = f(\rho^*)R^{*-4.4}_c \tag{31}$$

where

$$f(\rho^*) = 2.157\times10^{-7}\rho^{*-3.2}(1+0.0049\rho^*)^{4.13}. \tag{32}$$

A comparison of the predicted results with the experimental data values is presented in Fig. 4. Taking into account the variety of surfaces with different roughnesses used in these experiments, equation (31) allows a fairly good representation of the existing water data for a wide range of pressures from 1 to 198 bar. It must be noted here that the experimental data points of [25, 26] are based on the experimentally counted values of the active nucleation sites.

5. ACTIVE NUCLEATION SITE DENSITY IN CONVECTIVE NUCLEATE BOILING

5.1. *Site density correlation*

In some sense, bubble nucleation processes are similar in pool boiling and convective nucleate boiling. In both cases, to maintain nucleate boiling on a surface, it is necessary that the effective liquid superheat, $\Delta T_e \equiv T_g - T_{sat}$, exceeds a critical value for a specific system pressure. However, there is a significant difference between two cases in terms of the value of the effective superheat in which a bubble grows. A bubble nucleated at a center actually grows through a liquid

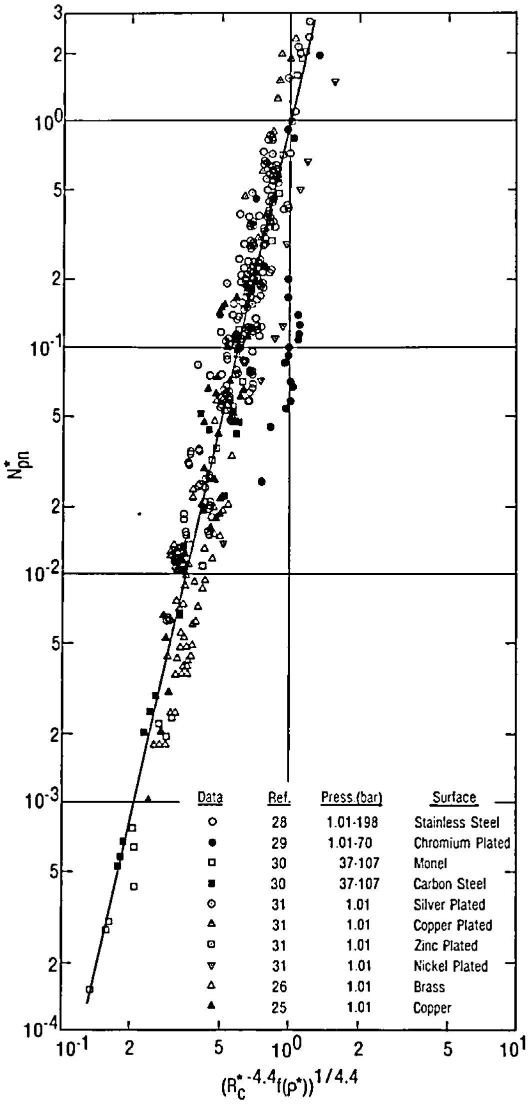

FIG. 4. Correlation of active nucleation site density in pool boiling.

film region adjacent to the wall where a considerably high temperature gradient exists, and so in reality it experiences a somewhat lower mean superheat than the wall superheat, $\Delta T_{sat} \equiv T_w - T_{sat}$. In the case of pool boiling, the difference is not significant, and the superheat based on the wall temperature can be taken as the effective superheat. In fact this is the superheat that has been used in the literature and in equation (29) for the pool boiling analysis. In the case of forced convective nucleate boiling, however, the temperature gradient is affected by the hydrodynamic flow field and would generally be much steeper than in the corresponding pool boiling case with the same wall superheat. Therefore, the effective superheat, in which a bubble grows in convective boiling, would be less than the actual wall superheat.

In view of these differences and the mechanistic similarity between pool and convective boiling, it is postulated here that the active nucleation site density correlation developed for pool boiling could be used also in the forced convective system by using an effective superheat, ΔT_e, rather than the actual wall superheat, ΔT_{sat}. Thus, the dimensionless active nucleation site density, N_{cn}, in the convective system can be expressed by

$$N^*_{cn} = R^{*-4.4}_{ce} f(\rho^*) \tag{33}$$

where $f(\rho^*)$ is defined by equation (32), and R_{ce} is the effective critical cavity size, which can be evaluated by equation (29) if the effective superheat is available.

A formal way of predicting the effective superheat requires information about the thermal boundary layer profile in the vicinity of the heated surface and the cavity size distribution. At the present time none of this information is known with sufficient accuracy. Instead, the concept of a suppressed factor is introduced here. In Chen's heat transfer correlation [32], which has been proved to be reliable in subcooled, and saturated nucleate boiling regions, a suppression factor S was defined by

$$S = (\Delta T_e/\Delta T_{sat})^{0.99}. \tag{34}$$

From experimental data it was graphically correlated to a two-phase flow Reynolds number defined by

$$Re_{TP} = [G(1-x)D/\mu_f]F^{1.25}. \tag{35}$$

Functional relationships which fit Chen's representations for F and S, respectively, are

$$F = 1.0 \text{ for } X_{tt} \geqslant 10,$$

$$\text{and} \quad F = 2.35(0.213 + 1/X_{tt})^{0.736} \text{ for } X_{tt} < 10 \tag{36}$$

$$\text{and} \quad S = 1/(1 + 1.5 \times 10^{-5} Re_{TP}) \tag{37}$$

where X_{tt} is the Martinelli parameter [33].

The ratio in equation (34) was arbitrarily taken to the 0.99 power by Chen in order that S may appear to the first power in his final heat transfer correlation. For simplicity the power 0.99 in equation (34) may be replaced by 1.0 and the effective superheat is calculated by

$$\Delta T_e = S \Delta T_{sat}. \tag{38}$$

With S and ΔT_e predicted, respectively, by equations (37) and (38), equation (33) can be used to predict the active nucleation site density in forced convective nucleate boiling.

Based on the foregoing development, the following qualitative observations can be made:

(1) Since the suppression factor and in turn the effective superheat decreases with increasing mass flow rate, the active nucleation site density decreases with increasing flow rate. This result has been confirmed by experimental observations. At relatively high mass flow rates nucleation may be completely suppressed.

(2) The number of active nucleation sites increases with increasing surface heat flux. This result is an

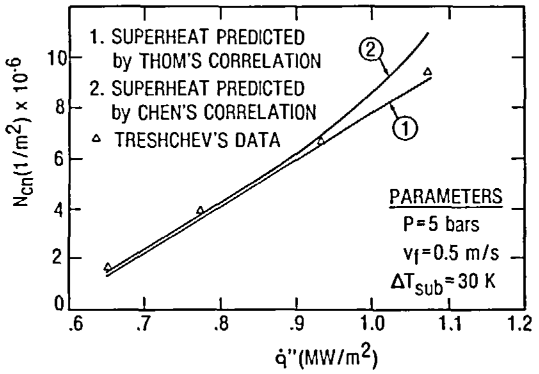

FIG. 5. Comparison of calculated active nucleation site density with Treshchev's convective boiling data [34].

expected one, because as the heat flux increases the wall superheat and, in turn, the effective superheat increases, indicating more and more nucleation sites are activated.

5.2. *Comparison*

In the case of the pool boiling there exists a sufficient number of experiments providing quantitative information about the active nucleation site density. However, in the case of forced convective nucleate boiling, so far there are very few experimental data [34] available which can provide such information. In this reference, the active nucleation site density was counted as a function of the surface heat flux. System pressure, average liquid velocity and the liquid subcooling were treated as parameters. Since the wall superheat, which is needed in the present correlation to evaluate the effective superheats, was not measured in Treshchev's experimental study [34], it is not possible to make a direct comparison. For the purpose of indirect comparisons, however, wall superheats are evaluated by Chen's [32] and Thom's [35] correlations, and the dimensional active nucleation site densities predicted by the method developed in the preceding section are compared with Treshchev's experimental observations in Figs. 5 and 6. In order to cover the whole range of experimental parameters, a complete comparison is presented in Fig. 7 in terms of the dimensionless variables.

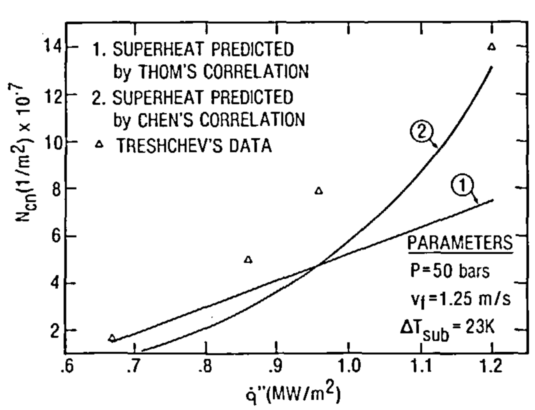

FIG. 6. Comparison of calculated active nucleation site density with Treshchev's convective boiling data [34].

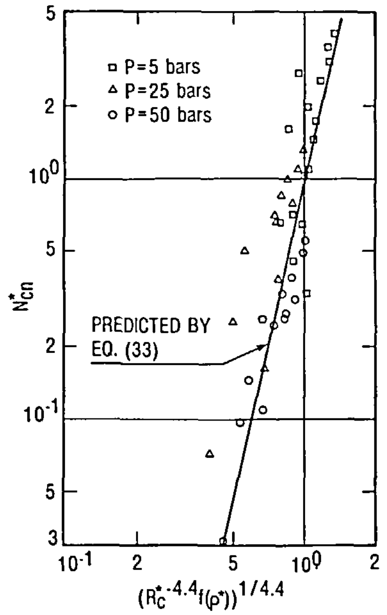

FIG. 7. Comparison of calculated dimensionless active nucleation site density with experimental convective boiling data [34].

Considering the approximations involved in these indirect comparisons, it is evident from these figures that the active nucleation site density predicted by equation (33) represents fairly well the only existing water data for a pressure range of 1–50 bar. In particular, wall temperatures based on Chen's correlation yield better agreement than that based on Thom's correlation. This may be due to the subcooling effects which were not taken into account by Thom's correlation.

6. SUMMARY AND CONCLUSIONS

(1) Realizing the significance of the interfacial area as an important parameter for predicting the interfacial transport terms in a two-fluid model formulation, the interfacial area concentration was expressed in terms of the kinematic parameters related to the structure of bubbly two-phase flow. These are the void fraction, bubble shape factor and the bubble number density.

(2) The bubble number density was formulated in terms of the number density differential transport equation which took into account the homogeneous and heterogeneous bulk liquid nucleation, wall cavity nucleation and the bubble collapse through the source and sink terms.

(3) The significance of these parameters in the heated boiling channel was discussed, and it was concluded that:

(a) for water at least, homogeneous bulk liquid nucleation could be discounted as a bubble source term,

(b) heterogeneous nucleation in the bulk liquid could be extremely important in the special situation such as the discharge of flashing steam–water mixtures through short nozzles or orifices. However, cavity nucleation is almost always an important source for bubbles in a system with heat addition.

(4) Based on the overall energy and mass balance equations, the bubble sink rate due to re-condensation was formulated in terms of cavity nucleation rate and the bulk liquid temperature gradient. The validity of this formulation was tested against two-limiting boundary conditions.

(5) Since the pressure ranges covered by the existing data on the nucleation site density were far short of being sufficient to arrive at a reliable correlation, quantitative site density data were obtained using the heat transfer correlation, which was developed by modeling the hydrodynamic field as a source flow around a growing bubble.

(6) The dimensionless active nucleation site density in pool boiling was correlated to the dimensionless minimum cavity size and the density ratio. It was shown that this type of correlation allowed a fairly good representation of the existing experimental water data for a pressure range of 1–198 bar.

(7) In view of the mechanistic similarity in bubble nucleation, it was postulated that the pool boiling active site density correlation could be used to predict the site density in the forced convective nucleate boiling with an effective liquid superheat rather than the actual wall superheat. The method was tested against the only subcooled nucleate boiling experimental data available in the literature. The good qualitative as well as quantitative agreement between experimentally measured active site density and those evaluated by the method offered here appeared to verify the basic principles involved in the development.

(8) Substituting equation (33) in equation (15), the bubble number density transport equation can be expressed as

$$\frac{\partial N_{\mathrm{b}}}{\partial t}+\frac{\partial}{\partial z}(N_{\mathrm{b}}v_{\mathrm{b}z})=\left(\frac{T_{\mathrm{f}}-T_{0}}{T_{\mathrm{sat}}-T_{0}}\right) \times\left[\frac{f(\rho^{*})}{R_{\mathrm{ce}}^{*4.4}D_{\mathrm{d}}^{2}}\right]\left(\frac{f\xi_{\mathrm{h}}}{A_{\mathrm{c}}}\right)+\phi_{\mathrm{he}}+\phi_{\mathrm{dis}} \quad (39)$$

where $f(\rho^{*})$ is defined by equation (32), and R_{ce}^{*} is the dimensionless minimum cavity size based on effective superheat.

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REFERENCES

1. M. Ishii, *Thermo-Fluid Dynamic Theory of Two-Phase Flow*. Eyerolles, Paris (1975).
2. P. Vernier and J. M. Delhaye, General two-phase flow equations applied to the thermodynamics of boiling nuclear reactor, *Energ. Prim.* 4(1) (1968).
3. J. Bour'e and M. Reocruex, General equations of two-phase flows, 4th All Union Heat Transfer Conference, Minsk, U.S.S.R. (1972).
4. G. Kocamustafaoguallari, Thermo-fluid dynamics of separated two-phase flow, Ph.D. Thesis, School of Mechanical Engineering, Georgia Institute of Technology, Georgia (1971).
5. N. Zuber, Flow excursions and oscillations in boiling, two-phase systems with heat addition, in *Proc. EUROTOM Symp. Two-Phase Flow Dynamics*, Vol. 1, pp. 1070–1089. Commission of European Communities, Brussels (1967).
6. M. Ishii, One dimensional drift-flux model and constitutive equations for relative motion between phases in various two-phase flow regimes, Argonne National Laboratory Report, ANL-77-47 (1977).
7. M. Ishii and T. C. Chawla, Two-fluid model and momentum interaction between phases, NRC 7th Water Reactor Safety Research Information Meeting, Gaithersburg (1979).
8. M. Ishii and K. Mishima, Two-fluid model and analysis of interfacial area, Argonne National Laboratory Report, ANL/RAS/LWR 80-3 (1980).
9. M. Volmer, *Kinetik der Phasenbildung*. Steinhopf, Leipzig; Edward Bros., Ann Arbor (1945).
10. R. J. Simoneau, Depressurization and two-phase flow of water containing high levels of dissolved nitrogen gas, NASA Technical Paper 1839 (1981).
11. R. W. Bowring, Physical model based on bubble detachment, and calculation of steam voidage in the subcooled region of a heated channel, Report Number HPR-10, Institute for Atomenergi, Halden, Norway (1962).
12. S. Levy, Forced convection subcooled boiling—Prediction of vapor volumetric fraction, *Int. J. Heat Mass Transfer* 10, 951–971 (1967).
13. S. Y. Ahmad, Forced convection subcooled boiling—prediction of the onset of bubble detachment, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada (1969).
14. P. Saha and N. Zuber, Point of net vapor generation and void fraction in subcooled boiling, 5th Int. Heat Transfer Conf. Tokyo, Paper B4.7 (1974).
15. G. Bijward, F. W. Staub and N. Zuber, A program of two-phase flow investigation, Eleventh Quarterly Report, October–December 1965, General Electric Co., San Jose, Ca., Report No. GEAP 5067, Euratom Report No. EURAEC 1575.
16. N. Zuber, F. W. Staub and G. Bijward, Vapor void fraction in subcooled boiling and saturated boiling systems, in *Proc. 3rd Int. Heat Transfer Conf.*, p. 24, A.I.Ch.E., New York (1966).
17. P. G. Kroeger and N. Zuber, An analysis of the effect of various parameters on the average void fractions in subcooled boilings, *Int. J. Heat Mass Transfer* **11**, 211–233 (1968).
18. P. Griffith and J. D. Wallis, The role of surface conditions in nucleate boiling, *Chem. Engng Prog. Symp. Ser.* 56, 49 (1968).
19. W. T. Brown, Study of flow surface boiling, Ph.D. Thesis, Mech. Engng Dept., M.I.T. (1967).
20. R. L. Judd and M. Shoukri, Nucleate boiling on oxide coated glass surface, *Trans. Am. Soc. Mech. Engrs*, Series C, *J. Heat Transfer* 97, 494–495 (1975).
21. K. Bier, D. Gorenflo, M. Salem and Y. Tames, Pool boiling heat transfer and size of active nucleation centers for horizontal plates with different surface roughness, *Proc. 6th Int. Heat Transfer Conf.*, Vol. 1, pp. 151–156, Toronto (1980).
22. G. Kocamustafaogullari, M. Ishii, and I. Y. Chen, Correlation for nucleation site density and its effect on

interfacial area, Argonne National Laboratory Report, ANL-82-32 (1982).

23. N. Zuber, Nucleate boiling. The region of isolated bubbles and the similarity with natural convection, *Int. J. Heat Mass Transfer* **6**, 53–78 (1963).
24. N. Zuber, Hydrodynamic aspects of nucleate pool boiling, Part-I The region of isolated bubbles, Rep. RQ-R1-164 (1960), AEC. Rep. T.I.D. 6338. Office Tech. Ser., Dept. of Com., Wash., D.C.
25. H. M. Kurihara and J. E. Myers, Fundamental factors affecting boiling, *Trans. Am. Inst. Chem. Engrs* **6**, 83 (1960).
26. K. Yamagata, F. Hirano, K. Nishikawa and H. Matsuoka, Nucleate boiling of water on a horizontal surface, *Mem. Fac. Engng, Kyushu Univ.* **15**, 97 (1955).
27. R. F. Gaertner, and J. W. Westwater, Population of active sites in nucleate boiling heat transfer, *Chem. Engng Prog. Symp. Ser.* **20**, 39–48 (1960).
28. V. Borishanskii, G. Bobrovich, and F. Minchenko, Heat transfer from a tube to water and to ethanol in nucleate pool boiling, in *Symposium of Heat Transfer and Hydraulics in Two-Phase Media* (edited by S. S. Kutateladze). Gosenergiodat, Moscow (1961).
29. T. M. Cichelli, and C. F. Bonilla, Heat transfer to boiling under pressure, *A.I.Ch.E. Jl* **41**, 775–787 (1945).
30. W. Elrod, J. Clark, E. Lady and H. Merte, Boiling heat transfer data at low heat flux, *Trans. Am. Soc. Mech. Engrs*, Series C, *J. Heat Transfer* **87**, 235–243 (1967).
31. U. Magrini and E. Nannei, On the influence of the thickness and thermal properties of heating walls on the heat transfer coefficients in nucleate pool boiling, *Trans. Am. Soc. Mech. Engrs*, Series C, *J. Heat Transfer* **97**, 173–178 (1975).
32. J. C. Chen, Correlation for boiling heat transfer to saturated fluids in convective flows, I/EC *Process Des. Dev.* **5**, 322 (1966).
33. R. C. Martinelli and D. B. Nelson, Prediction of pressure drop during forced circulation boiling of water, *Trans. Am. Soc. Mech. Engrs* **70**, 695–702 (1948).
34. G. G. Treshchev, The number of vapor-formation centers in surface boiling, in *Convective Heat Transfer in Two-Phase and One-Phase Flows* (edited by V. M. Borishonskii and I. T. Pallev). Translated from the Russian by the Israel Program for Scientific Translations (1969).
35. J. R. S. Thom, W. M. Walker, T. A. Fallon and G. F. S. Reising, Boiling in subcooled water during flow up heated tubes or annulis, *Proc. Symp. on Boiling Heat Transfer in Stream Generating Units and Heat Exchangers*, Manchester Inst. Mech. Engrs, London (1965).

AIRE INTERFACIALE ET DENSITE DE SITES DE NUCLEATION DANS LES SYSTEMES EN EBULLITION

Résumé—La densité numérique qui est un des plus importants paramètres affectant la concentration d'aire interfaciale dans un système diphasique en ébullition, est formulée à partir de l'équation de transport. Cette formulation rend apparent que la densité des sites actifs de nucléation sur une surface chauffée d'un canal est le paramètre fondamental pour prévoir la densité numérique des bulles. Une relation pour la densité des sites de nucléation est développée pour l'ébullition en réservoir et elle est étendue au cas de l'ébullition nucléée avec convection forcée. Les densités des sites calculées avec cette relation sont en accord relativement bon avec les mesures directes disponibles dans la littérature.

PHASENGRENZFLÄCHEN UND KEIMSTELLENDICHTE IN SIEDENDEN SYSTEMEN

Zusammenfassung—Die Keimstellendichte ist eine der wichtigsten Parameter, welche die Konzentration der Phasengrenzflächen in blasenförmiger Zweiphasenströmung beeinflussen. Sie wird in der Art der differentiellen Transportgleichung formuliert. Aus dieser Formulierung wird ersichtlich, daß die aktive Keimstellendichte an einer beheizten Kanaloberfläche der Schlüsselparameter zur Bestimmung der Blasendichte ist. Eine konstitutive Beziehung für die aktive Keimstellendichte wird für das Behältersieden entwickelt und auf das Blasensieden bei erzwungener Konvektion erweitert. Die aktive Keimstellendichte, die mit dieser konstitutiven Beziehung berechnet wird, stimmt gut mit direkten Messungen überein, wie sie in der Literatur zu finden sind.

ПЛОЩАДЬ ПОВЕРХНОСТИ РАЗДЕЛА И ПЛОТНОСТЬ ЦЕНТРОВ ПАРООБРАЗОВАНИЯ В КИПЯЩИХ СИСТЕМАХ

Аннотация—Плотность центров парообразования, которая является одним из наиболее важных параметров, определяющих концентрацию пузырьков на поверхности раздела в системе с двухфазным пузырьковым течением, описывается дифференциальным уравнением переноса. Такой подход показывает, что плотность активных центров парообразования на нагреваемой поверхности канала является основным параметром для определения плотности пузырьков. Предложено выражение для расчета плотности активных центров парообразования при кипении в большом объеме, обобщенное для пузырькового кипения при вынужденной конвекции. Плотность активных центров парообразования, рассчитанная по этому выражению, неплохо согласуется с опубликованными экспериментальными данными.